Density Control of Gaussian Mixtures Models: From Controlling Large Populations to Generative AI

Workshop on Stochastic Planning & Control of Dynamical Systems

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IEEE Control Systems Society™

Conference on Decision and Control
CDC2025
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Controlling the density of dynamical systems

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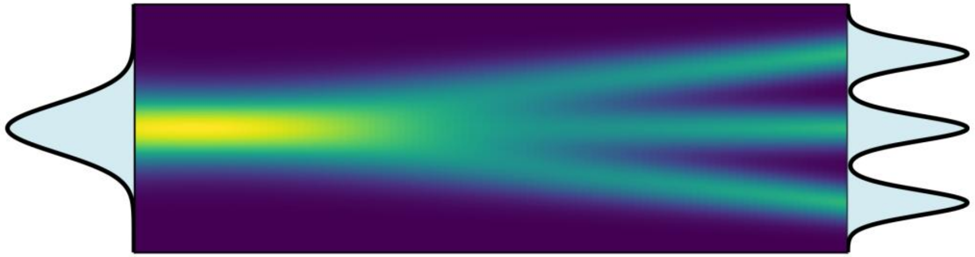
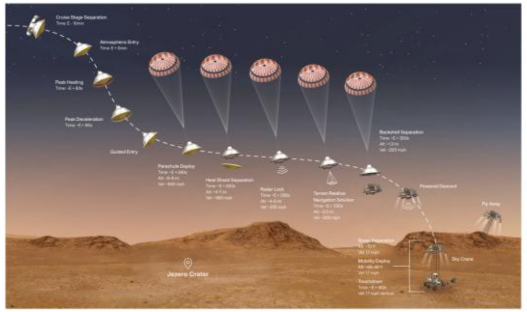
ChatGPT: Generate an image of a GT astronaut cat chasing a tennis ball.

$$\mathrm{d}x_t = u_t(x_t;\theta)\,\mathrm{d}t + \sqrt{\epsilon}\,\mathrm{d}w_t$$





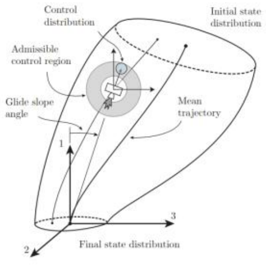
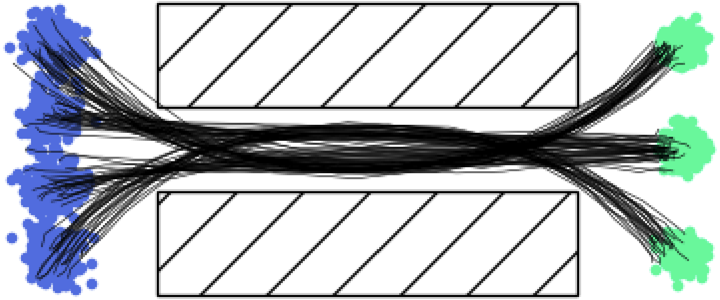


Figure 1: Powered descent with covariance control.

Ridderhof, Jack, and Panagiotis Tsiotras. "**Minimum-fuel powered descent in the presence of random disturbances.**" In *AIAA SciTech 2019 forum*, p. 0646. 2019.



George Rapakoulias, Ali Reza Pedra, Panagiotis Tsiotras. **"Steering large agent populations using mean field Schrodinger Bridges and Gaussian mixture models".** Control Systems letters 2025.

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Schrödinger Bridge and Flow Matching

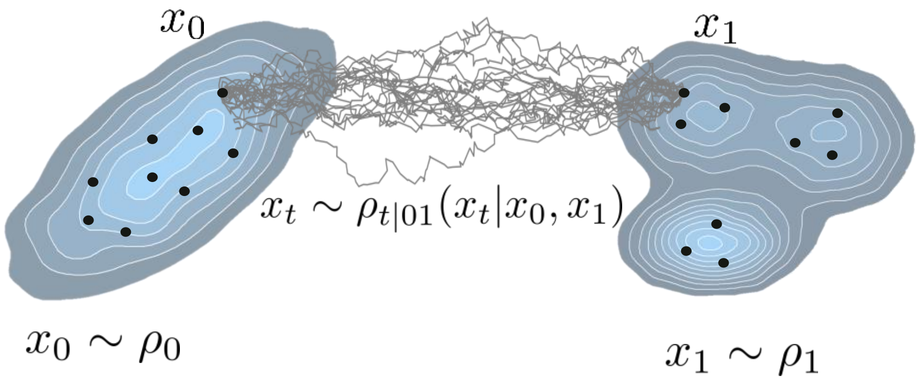
Diffusion Schrödinger Bridge Matching: "Optimal flow" = "Conditional flows" + "Transport plan" [1-4]

Schrödinger Bridge

$$\min_{\rho,u} \mathbb{E}\left[\int \|u_t(x_t)\|^2 \, \mathrm{d}t\right],$$
$$\text{s.t. } \mathrm{d}x_t = u_t(x_t)\,\mathrm{d}t + \sqrt{\epsilon}\,\mathrm{d}w,$$
$$x_0 \sim \rho_0,$$
$$x_1 \sim \rho_1$$

=

Conditional Bridge



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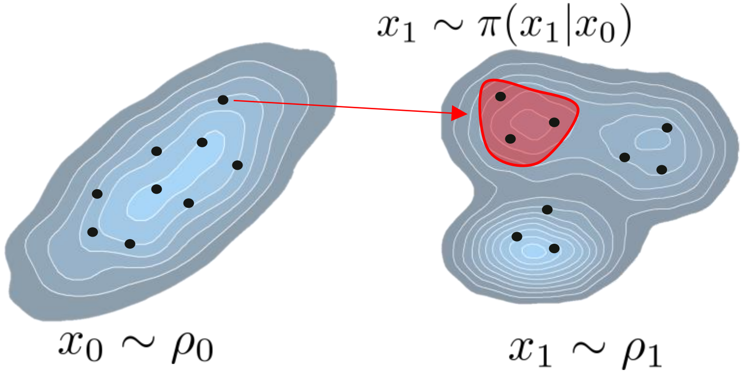
Transport plan



Solution is a mixture of elementary solutions

$$x_t \sim \rho_t = \int \rho_{t|01}(x_t)\mathrm{d}\pi, \quad u_t(x_t) = \int u_{t|01}(x_t)\frac{\rho_{t|01}(x_t)}{\rho_t(x_t)}\mathrm{d}\pi$$

[1] Shi., et al. "Diffusion Schrödinger bridge matching." (NeurIPS 2023).
[2] Peluchetti, Stefano. "Diffusion bridge mixture transports, Schrödinger bridge problems and generative modeling." JMLR (2023).
[3] Y. Lipman, et al. "Flow Matching for Generative Modeling." (ICLR 2023).
[4] Terpin, A., et al. "Dynamic programming in probability spaces via optimal transport." SIAM Journal on Control and Optimization 62.2 (2024)

Schrödinger Bridges for Gaussian Mixtures

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Can we use find settings where this decomposition yields tractable mixtures?

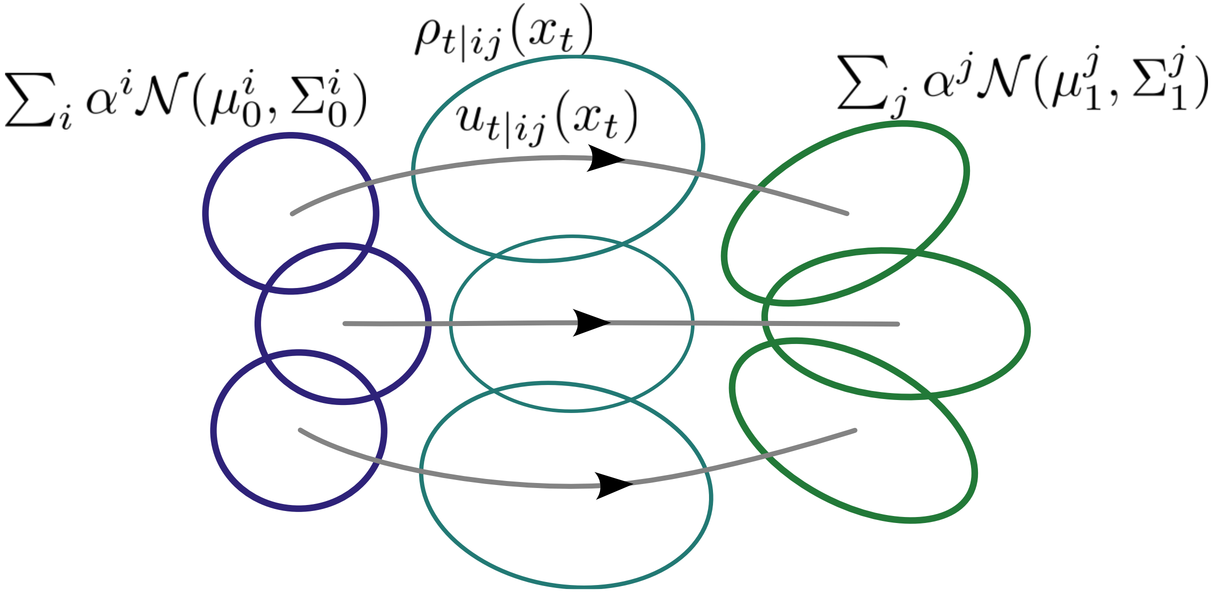
GMM SB

$$\min_u \quad J = \mathbb{E}\left[\int_0^1 \|u_t(x_t)\|^2 \mathrm{d}t\right],$$

$$\text{s.t.} \quad \mathrm{d}x_t = u_t(x_t)\,\mathrm{d}t + \sqrt{\epsilon}\,\mathrm{d}t$$

$$x_0 \sim \textstyle\sum_{i=1}^{N_0} \alpha_0^i \mathcal{N}(\mu_i^0, \Sigma_i^0),$$

$$x_1 \sim \textstyle\sum_{j=1}^{N_1} \alpha_1^j \mathcal{N}(\mu_j^1, \Sigma_j^1)$$



<u>Key intuition:</u> Define a feasible set of solutions as a **mixture of Gaussian Schrödinger Bridges.**

Family of feasible policies parametrized by λ_{ij}

$$\rho_t(x) = \textstyle\sum_{ij} \lambda_{ij} \rho_{t|ij}(x)$$

$$u_t(x) = \textstyle\sum_{i,j} u_{t|ij}(x) \frac{\rho_{t|ij}(x)\lambda_{ij}}{\sum_{i,j} \rho_{t|ij}(x)\lambda_{ij}}$$

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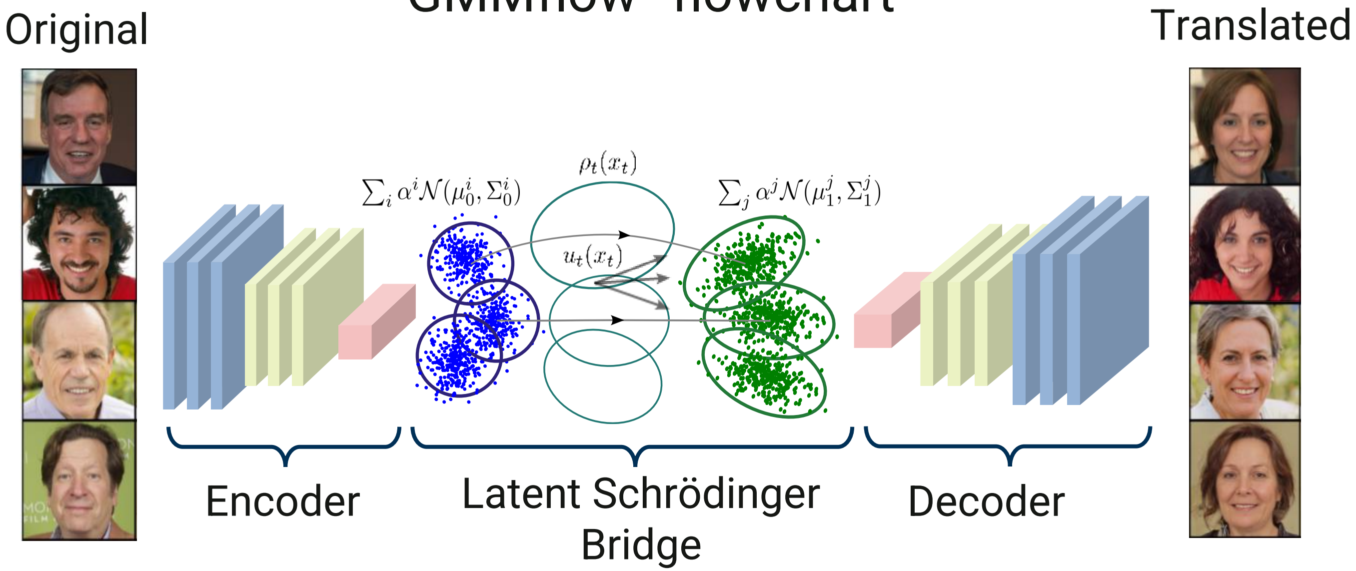
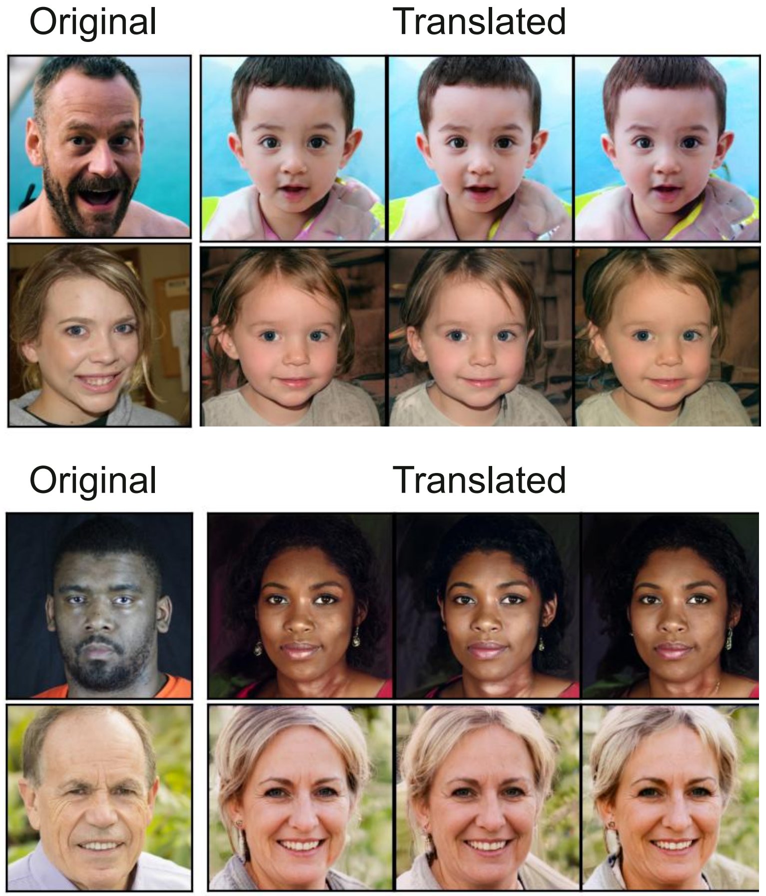
Image translation

GMMflow[1] flowchart





Implementation details:

- ALAE[2] autoencoder (512-mensional latent space)
- 10-component mixture in each class.
- **Few seconds for training, sub-second inference time**!

[1]Rapakoulias G. et al. "**Go with the flow: Fast diffusion for Gaussian Mixture Models.**" NeurIPS (2025)
[2]Pidhorskyi et al. "**Adversarial latent autoencoders**." *IEEE/CVF* (2020).

Swarm control as a density control problem

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Can we use the GMM-SB framework for swarm control?

Given the **agent-level dynamics** and the **interaction law between agents**, construct a system whose density captures the evolution of the swarm[1,2]?

Agent dynamics $\mathrm{d}x_t = -\nabla V(x_t)\mathrm{d}t + \mathrm{d}w_t$

Finite swarm $\left\{\mathrm{d}x_t^j = -\nabla V(x_t^j)\mathrm{d}t - \frac{1}{N-1}\sum_{i\neq j}\nabla W(x_t^j - x_t^i) + \mathrm{d}w_t^i, \quad j = 1, \ldots N, \right\}$

Mean-field system $\mathrm{d}x_t = -\nabla V(x_t)\mathrm{d}t - \int \nabla W(x_t - y)\rho_t(y)\mathrm{d}y + \mathrm{d}w_t$

[1] Chen, Yongxin. "Density control of interacting agent systems." *IEEE Transactions on Automatic Control* 69.1 (2023): 246-260.
[2] Elamvazhuthi, Karthik, and Spring Berman. "Mean-field models in swarm robotics: A survey." *Bioinspiration & Biomimetics* 15.1 (2019): 015001.

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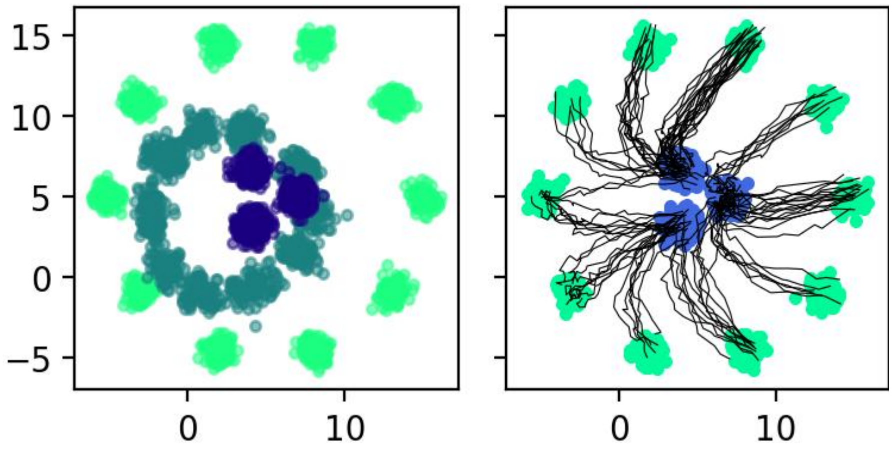
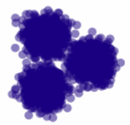
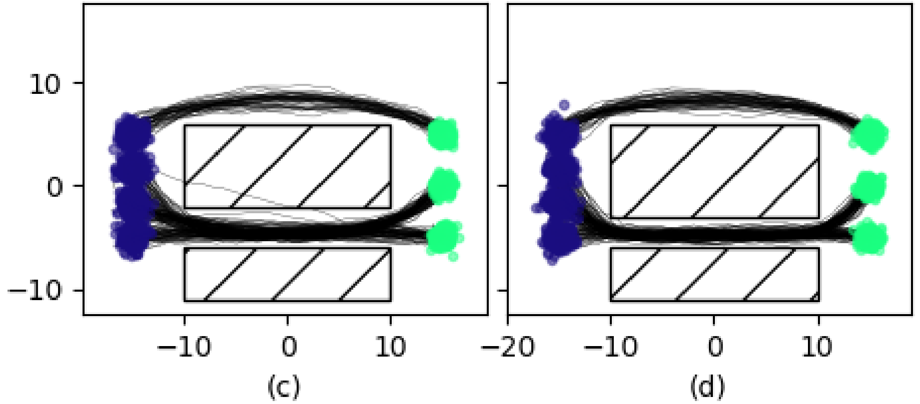
GMM mean field examples

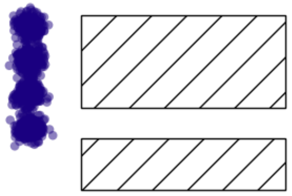
Unconstrained Mean-Field problem:





Constrained Mean-Field problems:





Thank you!